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# Nano $\hbar_{\alpha}$ - open set in Nano Topological Spaces

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**ABSTRACT**

**The primary purpose of this paper is to introduce a new classification of Nano open sets, specifically Nano $\hbar_{\alpha}$- open set. Some results have been established regarding the characterization of these newly generalized sets that serve as the foundation for the definition of Nano continuous mappings. In addition, the concept of Nano $\hbar_{\alpha}$-open mappings, continuous functions, and Nano $\hbar_{\alpha}$- homeomorphism has been proposed. The investigation of properties related to these functions has yielded some remarks that have been supported by examples.**

***Keywords***- Nano topological space, Nano $\hbar$ - open set, Nano $\hbar_{\alpha}$- open set, Nano $\hbar_{\alpha}$– continuous mapping, Nano $\hbar_{\alpha}$– homeomorphism mapping.

## I. INTRODUCTION

In 2013, Thivagar [5] introduced the concept of Nano topological space as an approximation space and boundary region of the universe set Ư using Ɽ be an equivalence relation. The pair (Ư, Ɽ) is said to be the approximation space, Where Ӿ ⊆ Ư so,

**1.** $Ⱡ_{Ɽ}$ (Ӿ) = $\cup_{\varkappa \in Ư}${Ɽ (ϰ): Ɽ (ϰ)) ⊆ Ӿ} is lower approximation of Ӿ with respect to Ɽ, where Ɽ (ϰ) denotes the equivalence class determined by Ӿ.

**2.** $\mathring{U}_{Ɽ}$ (Ӿ) = $\cup_{\varkappa \in Ư}${ Ɽ (ϰ): Ɽ (ϰ) ∩ Ӿ≠ $\varphi$ } is Upper approximation of Ӿ with respect to Ɽ.

**3.** $Ƀ_{Ɽ}$ (Ӿ)= $\mathring{U}_{Ɽ}$ (Ӿ) - $Ⱡ_{Ɽ}$ (Ӿ) is boundary region of Ӿ with respect to Ɽ.

And he investigated the characteristics of it in [1].

Then, $Ƭ_{Ɽ}$ (Ӿ)= {Ư, φ, $Ⱡ_{Ɽ}$ (Ӿ), $\mathring{U}_{Ɽ}$ (Ӿ), $Ƀ_{Ɽ}$ (Ӿ)} is called a Nano topology Complies with the following axioms:

1.Ư and φ ∈$Ƭ_{Ɽ}$ (Ӿ).

2.The union of the elements of any sub collection of $Ƭ_{Ɽ}$ (Ӿ) is in $Ƭ_{Ɽ}$ (Ӿ).

3. The intersection of the elements of any finite sub collection of $Ƭ_{Ɽ}$ (Ӿ) is in $Ƭ_{Ɽ}$ (Ӿ).

The pair (Ư, $Ƭ_{Ɽ}$ (Ӿ)) is said to be Nano topological space (ꞑƬ$\mathcal{S}$). Thus, every element of $Ƭ_{Ɽ}$ (Ӿ) is called Nano- open set (ꞑ-*os*) and the complement is said to be Nano closed set (ꞑ-*cs*), the set Ɓ = {Ư, $Ⱡ_{Ɽ}$ (Ӿ), $Ƀ_{Ɽ}$ (Ӿ)} is the base for ꞑƬ$\mathcal{S}$. [5]

Also he defined Nano α open set(ꞑα -os), if Ą⊆ Ư such that Ą ⊆ ꞑint (ꞑCl (ꞑint (Ą))) so the set of all ꞑα -os is denoted by $Ƭ_{Ɽ}^{\alpha}$ (Ӿ) and proved theorem every ꞑ - *os* in (Ư, $Ƭ_{Ɽ}$ (Ӿ)) is (ꞑα -os) in Ư. But convers is not need to be true [5].

In 2013, he introduced the concept of Nano continuous (ꞑ- *cont*). on Ư. Where he showed it through the mapping ꞙ: (Ư, $Ƭ_{Ɽ}$ (Ӿ)) → (V, $Ƭ_{Ɽ}$` (Ч)) is said to be Nano continuous on Ư if the inverse image of each ꞑ-*os* in V is ꞑ-*os* in Ư [4].

Sankar, P. K. (2019) studied the concept of Nano totally continuous functions in Nano topological space.[2]

## II. PRELIMINARIES

**Definition 2.1 [3]** A space (Ư, $Ƭ_{Ɽ}$ (Ӿ)) is ꞑƬ$\mathcal{S}$ , let Ą⊆Ư, then Ą be Nano $\hbar$ -open (ꞑ$\hbar$ - os) if Ą ⊆ ꞑInt (Ą ∪ O) for every O non empty ꞑ- *os* in Ư and O ≠ Ư. The

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complement of (ꞍℏN- os) is Nano $\hbar$ -closed set (Ꞥ$\hbar$ -cs) and the set of all Ꞥ$\hbar$ - *os* in (Ư, Ƭ$_R$ (Ӿ)) is denoted by$Ƭ_R^{\hbar}$ (Ӿ).

We conclusion that the (Ꞥ$\hbar$ -os) and (Ꞥα -os) are independent.

**Theorem 2.2 [3]** Every Ꞥ - *os* Ą in ꞤƬ$\mathcal{S}$ is Ꞥ$\hbar$ -os. But the convers is not necessary to be true.

**Definition 2.3 [3]** Let ₣: (Ư, Ƭ$_R$ (Ӿ)) → (Ѵ, Ƭ$_R$ ` (Ч)) is said to be Nano $\hbar$ continuous if ₣$^{-1}$ (Ą) is Ꞥ$\hbar$ -os in Ư for every Ꞥ-os Ą in Ѵ and is denoted by (Ꞥ$\hbar$ *-cont*).

**Example 2.4** let ₣: (Ư, Ƭ$_R$ (Ӿ)) → (Ѵ, Ƭ$_R$` (Ч)) , Ư= Ѵ = {c,e,d}, Ư/Ꞧ ={{c},{e},{d}} and let Ӿ ={c,d}, we get Ƭ$_R$ (Ӿ) ={ φ, Ư, {c,d}},$Ƭ_R^{\hbar}$ (Ӿ) = Þ(Ư) and let Ч={e,d}, Ѵ/Ꞧ={{c,e},{d}}, we get Ƭ$_R$` (Ч) ={ φ, Ư,{d},{c,e}}. Define the mapping by ₣(c) = e, ₣(e) = c, ₣(d) = d, Thus, ₣ is Ꞥ$\hbar$ *-cont* mapping.

**Theorem 2.5 [3]** Every Ꞥ*-cont* mapping Ꞥ$\hbar$ *-cont* mapping. But the convers is not necessary to be true. As the example (2.4) is Ꞥ$\hbar$ *-cont* mapping but it is not Ꞥ-*cont* (because ₣$^{-1}$({d}) ={d} ∉ Ƭ$_R$ (Ӿ)).

**Definition 2.6 [3]** Consider two Nano topological spaces (Ư, Ƭ$_R$ (Ӿ)) and (Ѵ, Ƭ$_R$` (Ч)) and Ą ⊆ Ư. Next the mapping ₣: (Ư, Ƭ$_R$ (Ӿ)) → (Ѵ, Ƭ$_R$` (Ч)) is called Nano$\hbar$ - totally continuous (ꞤΓ$\hbar$ $-cont$ ) if ₣$^{-1}$ (Ą) is Nano clopen in Ư for every Ꞥ$\hbar$ - os in Ѵ.

**Theorem 2.7 [3]**A mapping ₣: (Ư, Ƭ$_R$ (Ӿ)) → (Ѵ, Ƭ$_R$ `(Ч)) is ꞤΓ$\hbar$ $-cont$ ,only if ₣ is ꞤΓ *-cont*. The Converse of this theorem is not necessary to be true.

**Remark 2.8** The two mappings Ꞥ$\hbar$ *-cont* and ꞤΓ$\hbar$ $-cont$ are independent mappings.

**Definition 2.9[3]**A mapping ₣: (Ư, Ƭ$_R$ (Ӿ)) → (Ѵ, Ƭ$_R$`(Ч)), is said to be:

**1.** Nano $\hbar$ - open mapping (Ꞥ$\hbar$ *-OM)* if ₣(Ą) is Ꞥ$\hbar$ - os in Ѵ for every Ꞥ*-os* Ą in Ư.

**2.** Nano $\hbar$ - closed mapping (Ꞥ$\hbar$ *-CM* ) if ₣(Ą) is Ꞥ$\hbar$ *-cs* in Ѵ for every Ꞥ*-cs* Ą in Ư.

**Definition 2.10 [3]** Let ₣: (Ư, Ƭ$_R$ (Ӿ)) → (Ѵ, Ƭ$_R$ `(Ч)) is said to be a one-one and onto. Then ₣ is a Nano $\hbar$ - homeomorphism if and only if ₣ is both Ꞥ$\hbar$ *-cont* and Ꞥ$\hbar$ -*OM.*

## III. ON NANO $\hbar_{\alpha}$ - OPEN SETS

Our focus is on explaining the concept of a Nano $\hbar_\alpha$ - open set, examining its properties, and analyzing its relationes.

**Definition 3.1** Let (Ư, Ƭ$_R$ (Ӿ)) be ꞤƬ$\mathcal{S}$ and Ą a subset of Ư. Then Ą is said to be Nano $\hbar_{\alpha}$ – open set (Ꞥ$\hbar_{\alpha}$ - *os*) if Ą ⊆ Ꞥint$_\alpha$ (Ą ∪ Ơ) for every Ơ non empty Nano α– open set in Ư, where Ơ ≠ Ư.

**Remarks 3.2**

**1.** A subset Ą of (Ư, Ƭ$_R$ (Ӿ)). If Ą is Ꞥ$\hbar_{\alpha}$ - *os*, then the complement of Ą is said to be Nano $\hbar_{\alpha}$ – closed set (Ꞥ$\hbar_{\alpha}$ - *cs*).

**2.** The family of all Ꞥ$\hbar_{\alpha}$ - *os* (Ꞥ$\hbar_{\alpha}$ - *cs*) subsets of (Ư, Ƭ$_R$ (Ӿ)) will be denoted by Ꞥ$\hbar_{\alpha}$ O(Ư, Ӿ), (Ꞥ$\hbar_{\alpha}$ C(Ư, Ӿ)).

**3.** A subset Ą of space (Ư, Ƭ$_R$ (Ӿ)) is said to be Nano $\hbar_{\alpha}$ – clopen (Ꞥ$\hbar_{\alpha}$ - clopen) set if it is both Ꞥ$\hbar_{\alpha}$ - *os* and Ꞥ$\hbar_{\alpha}$ *-cs* sets.

**Example 3.3** Let Ư={a,b,c,d}, where Ư/ Ꞧ ={{a},{b, c} ,{d}} and Ӿ={a}. So, Ƭ$_R$ (Ӿ) = {φ,Ư,{a}} and $Ƭ_R^{\hbar}$ (Ӿ) ={φ, Ư,{a},{b,c,d}}.

We get $Ƭ_R^{\alpha}$ (Ӿ) ={ φ, Ư, {a},{a,b}, {a,c},{a,d}, {a,b,c}, {a,b,d},{a,c,d}} and Ꞥ$\hbar_{\alpha}$ O(Ư, Ӿ) = Þ(Ư).

**Remark 3.4**

**1.**The indiscrete ꞤƬ$\mathcal{S}$. doesn't have open sets of Ꞥ$\hbar_{\alpha}$ --type.

**2.**The collection of all Ꞥ$\hbar_{\alpha}$ - *o* sets in (Ư, Ƭ$_R$ (Ӿ)) is denoted by $Ƭ_R^{\hbar_\alpha}$ (Ӿ) .

**Propositions 3.5**

**1.**Every Ꞥ$\hbar$ - *os* in any ꞤƬ$\mathcal{S}$ (Ư, Ƭ$_R$ (Ӿ)) is Ꞥ$\hbar_{\alpha}$ - os.

**Proof.** A subset Ą of space (Ư, Ƭ$_R$ (Ӿ)) and Ą is Ꞥ$\hbar$ - *os* in Ư, by definition (3.1) we get Ą ⊆ Ꞥint$_\alpha$ (Ą∪ Ơ) and by using theorem (Every Ꞥ - *os* is Ꞥα-*os*). Thus, Ą is Ꞥ$\hbar_{\alpha}$ - *os* in Ư.

**2.** Every Ꞥ - *os* in any (Ư, Ƭ$_R$ (Ӿ)) is Ꞥ$\hbar_{\alpha}$ - *os.*

**Proof.** Let Ą⊆ Ư be Ꞥ*-os* in Ư, then Ą is Ꞥ$\hbar$ - *os* by using theorem (2.2). Since ∀ Ꞥ$\hbar$ - *os* is Ꞥ$\hbar_{\alpha}$ - *os* from part (1). Thus, Ą is Ꞥ$\hbar_{\alpha}$ - *os.*

**3.** Every Ꞥα - *os* in any ꞤƬS (Ư, Ƭ$_R$ (Ӿ)) is Ꞥ$\hbar_{\alpha}$ - *os.*

**Proof.** Let Ą⊆Ư be Ꞥα - *os*, so Ą = Ꞥint$_\alpha$ (Ą) by using definition (3.1). Then Ą is Ꞥ$\hbar_\alpha$ - *os.*

**Remark 3.6** The converse of the propositions (3.5) is not necessary to be true. By the example (3.3), we can find that {c, d} is Ꞥ$\hbar_{\alpha}$ -os but it is not (Ꞥ$\hbar$ - *os,* Ꞥ – *os* and Ꞥα – *os*).

**Definition 3.7** A space (Ư, Ƭ$_R$ (Ӿ)) is ꞤƬ$\mathcal{S}$ with respect to Ӿ, let Ą⊆ Ư is said to be Nano$\hbar_\alpha$ - interior of Ą (Ꞥint$\hbar_\alpha$ (Ą)) is defined as union of all Ꞥ$\hbar_\alpha$ - open subsets contained in Ą ,Ꞥint$\hbar_\alpha$ (Ą) is the largest Ꞥ$\hbar_\alpha$ - open subset of Ą.

**Example 3.8** Let Ư={a,b,c,d}, Ӿ={a,b}, where Ư/Ꞧ ={{a},{b, c},{d}}. So, Ƭ$_R$ (Ӿ)= {φ, Ư,{a},{a,b,c},{b,c}}, then$Ƭ_R^{\hbar_\alpha}$ (Ӿ) ={ φ, Ư,{a},{b,c},{a,b,c},{b,c,d}} . Let Ą= {a, b} , we get Ꞥin$\hbar_\alpha$ (Ą) ={a}.

**Definition 3.9** A space (Ư, Ƭ$_R$ (Ӿ)) is ꞤƬ$\mathcal{S}$ with respect to Ӿ, let Ą⊆ Ư. The set of all Ꞥ$\hbar_{\alpha}$ -cluster points of a subset Ą of Ư is said to be Ꞥ$\hbar_{\alpha}$ -closure of Ą and denoted by (ꞤCl$\hbar_{\alpha}$ (Ą)) and ꞤCl$\hbar_\alpha$ (Ą) is the smallest Ꞥ$\hbar_\alpha$ - *cs* of Ą.

Equivalently, ꞤCl$\hbar_{\alpha}$ (Ą) is the intersection of all Ꞥ$\hbar_{\alpha}$ - *cs* containing Ą. From the example (3.8), let Ą={a,b,c} . Thus ꞤCl$\hbar_{\alpha}$ (Ą) = {a,b,c}.

**Propositions 3.10** The space (Ư, Ƭ$_R$ (Ӿ)), then:

1. ꞤCl$\hbar_{\alpha}$ ( $B_R\hbar_{\alpha}$ (Ӿ)) =Ủ$_R\hbar_{\alpha}$ (Ӿ$^c$).
2. ꞤCl$\hbar_{\alpha}$ (Ƚ$_R\hbar_{\alpha}$ (Ӿ)) = Ƚ$_R\hbar_{\alpha}$ (Ӿ$^c$).
3. ꞤCl$\hbar_{\alpha}$ (Ủ$_R\hbar_{\alpha}$ (Ӿ)) = Ư.

**Proof.**

If φ, Ư, Ƚ$_R\hbar_{\alpha}$ (Ӿ), Ủ$_R\hbar_{\alpha}$ (Ӿ) and $B_R\hbar_{\alpha}$ (Ӿ) are Ꞥ$\hbar_{\alpha}$ - o sets in (Ư, Ƭ$_R$ (Ӿ))) then φ, Ư, ( Ƚ$_R\hbar_{\alpha}$ (Ӿ))$^c$, (Ủ$_R\hbar_{\alpha}$ (Ӿ))$^c$ and ($B_R\hbar_{\alpha}$ (Ӿ))$^c$ are Ꞥ$\hbar_{\alpha}$ - c sets in (Ư, Ƭ$_R$ (Ӿ)).

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Where $(\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c = \text{Ł}_R\hbar_\alpha\ (\text{Ӿ}^c)$ , $(\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}))^c = \text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}^c)$ and $(B_R\hbar_\alpha\ (\text{Ӿ}))^c = \text{Ł}_R\hbar_\alpha\ (\text{Ӿ}) \cup \text{Ł}_R\hbar_\alpha\ (\text{Ӿ}^c)$.

**1**. $B_R\hbar_\alpha\ (\text{Ӿ}) \subseteq \text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})$ and( $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}))^c \cap B_R\hbar_\alpha\ (\text{Ӿ}) \neq \varphi$, that is cannot contain $B_R\hbar_\alpha\ (\text{Ӿ})$, $(B_R\hbar_\alpha\ (\text{Ӿ}))^c$ are non – empty, that is $(\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c$ unless $B_R\hbar_\alpha\ (\text{Ӿ}) = \varphi$ ,but $(\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}))^c$ is ꟽ$\hbar_\alpha$ - *cs* containing $B_R\hbar_\alpha\ (\text{Ӿ})$. Since ꟽCl$\hbar_\alpha\ (B_R\hbar_\alpha\ (\text{Ӿ})) = (\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}))^c$, then ꟽCl$\hbar_\alpha$ ( $B_R\hbar_\alpha\ (\text{Ӿ})) = \text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}^c)$.

**2.** $(\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}))^c$ and $(\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c$ cannot contain $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ})$, if $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}) = \varphi$ , $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}) \cap (\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c = \varphi$ , then $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}) \subseteq \text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})$. In which case, ꟽCl$\hbar_\alpha$ $(\text{Ł}_R\hbar_\alpha\ (\text{Ӿ})) = (B_R\hbar_\alpha\ (\text{Ӿ}))^c$ , moreover it $(B_R\hbar_\alpha\ (\text{Ӿ}))^c$ and Ủ are ꟽ$\hbar_\alpha$ - c sets containing $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ})$. Then ꟽCl$\hbar_\alpha$ $(\text{Ł}_R\hbar_\alpha\ (\text{Ӿ})) = \text{Ł}_R\hbar_\alpha\ (\text{Ӿ}^c)$.

**3**. $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}) \subseteq (\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}))^c$ ,if $\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}) \subseteq (\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c$ .Then $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}) = \varphi$, and $\text{Ƭ}_R^{\hbar_\alpha}\ (\text{Ӿ}) = \{\varphi, \text{Ủ}, \text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})\}$, so $\varphi$, Ủ, $(\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c$ are ꟽ$\hbar_\alpha$ - c sets in Ủ .We get Ủ is ꟽ$\hbar_\alpha$ - c set containing $\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})$.

If $\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}) \subseteq (\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}))^c$ , then ꟽCl$\hbar_\alpha$ $(\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})) = \text{Ủ}$. If $\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}) \subseteq (B_R\hbar_\alpha\ (\text{Ӿ}))^c$ ,then $B_R\hbar_\alpha\ (\text{Ӿ}) \subseteq (\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c \subseteq (B_R\hbar_\alpha\ (\text{Ӿ}))^c$ . Since $B_R\hbar_\alpha\ (\text{Ӿ}) = \varphi$. Therefore, $\text{Ł}_R\hbar_\alpha\ (\text{Ӿ}) = \text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})$ , we get $\text{Ƭ}_R^{\hbar_\alpha}\ (\text{Ӿ}) = \{\varphi$, Ủ, $\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})\}$ and $\varphi$, Ủ,( $\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ}))^c$ are ꟽ$\hbar_\alpha$ - c sets in Ủ, then ꟽCl$\hbar_\alpha$ $(\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})) = \text{Ủ}$.

Since Ủ is the only ꟽ$\hbar_\alpha$ - *cs* containing $\text{Ủ}_R\hbar_\alpha$ (Ӿ), thus ꟽCl$\hbar_\alpha$ $(\text{Ủ}_R\hbar_\alpha\ (\text{Ӿ})) = \text{Ủ}$ are both cases.

**Definition 3.11** A space (Ủ, $\text{Ƭ}_R$ (Ӿ)) is ꟽƬ$\mathcal{S}$ , let Ą⊆ Ủ be Nano $\hbar_\alpha$ - exterior ( ꟽ$\mathcal{E}xt\hbar_\alpha$ (Ą)) if ꟽ$\mathcal{E}xt\hbar_\alpha$ (Ą) =ꟽint$\hbar_\alpha$ $(\text{Ą}^c)$.

From the example (3.8), let Ą={a}, $\text{Ą}^c$ ={b,c,d}. Thus ꟽ$\mathcal{E}xt\hbar_\alpha$ (Ą) =ꟽint$\hbar_\alpha$ ({b,c,d}) ={b,c,d}.

**Some of the properties of the sets' exterior are**

1. ꟽ$\mathcal{E}xt\hbar_\alpha$ (Ủ) = φ , ꟽ$\mathcal{E}xt\hbar_\alpha$ (φ ) = Ủ.
2. ꟽ$\mathcal{E}xt\hbar_\alpha$ (Ą) is ꟽ$\hbar_\alpha$ - open set.
3. ꟽ$\mathcal{E}xt\hbar_\alpha$ (Ą) = Ủ \ ꟽCl$\hbar_\alpha$ (Ą).

**Definition 3.12**

Let (Ủ, $\text{Ƭ}_R$ (Ӿ)) is ꟽƬS and Ą⊆ Ủ. A point ϰ ∈ Ủ is said to be ꟽ$\hbar_\alpha$ -limit point of Ą (ꟽ$\ell^{\hbar_\alpha}$ (Ą)), if it satisfies the following assertion:

$$(\forall G \in \text{Ƭ}_R^{\hbar_\alpha}(\text{Ӿ})), (\forall \varkappa \in G \Rightarrow G \cap (\text{Ą} - \{\varkappa\}) \neq \varphi)$$

The set of ꟽ$\hbar_\alpha$ - limit points of Ą is said to be the derived set of Ą (ꟽ$\mathcal{D}\hbar_\alpha$ (Ą)).

**Some of the properties of the derived set are**

**1.** ꟽCl$\hbar_\alpha$ (Ą) = (Ą ∪ ꟽ$\mathcal{D}\hbar_\alpha$ (Ą)).

**2.** Ą is ꟽ$\hbar_\alpha$ *-cs* if and only if ꟽ$\mathcal{D}\hbar_\alpha$ (Ą)) ⊆Ą.

**3.** ꟽ$\mathcal{D}\hbar_\alpha$ (Ą)) ⊆ꟽCl (Ą).

**Example 3.13**

Let Ủ ={2,3,4}, Ủ/ Ʀ ={{2},{3},{4}}and Ӿ={2,3}. Let Ą={2,3} , so $\text{Ƭ}_R$ (Ӿ) = { φ, Ủ,{2,3}},$\text{Ƭ}_R^{\hbar_\alpha}$ (Ӿ) = { φ, Ủ,{2},{3},{4},{2,3},{2,4},{3,4}}, then we get ꟽ$\mathcal{D}\hbar_\alpha$ (Ą) = φ .

**Remark 3.14** If Ą is a subset of Discreet $\text{Ƭ}_R^{\hbar_\alpha}$ (Ӿ), then ꟽ$\mathcal{D}\hbar_\alpha$ (Ą) = φ.

**Theorem 3.15**

If Ą is a singleton subset of $\text{Ƭ}_R^{\hbar_\alpha}$ (Ӿ), then ꟽ$\mathcal{D}\hbar_\alpha$ (Ą) = ꟽCl$\hbar_\alpha$ (Ą) – Ą.

**Proof.**

Since Ą is singleton subset of Ủ. Either if ϰ ∈ Ą and Ą ={ϰ} then G ∩ (Ą − {ϰ}) = φ. Or if ϰ ∉ Ą , then ꟽ$\hbar_\alpha$ - *os* G ∋ ϰ ∈ G and G ∩ (Ą − {ϰ}) ≠ φ,then ϰ ∈ ꟽD$\hbar_\alpha$ (Ą) . But, ꟽD$\hbar_\alpha$ (Ą) ⊆ ꟽCl$\hbar_\alpha$ . We get ϰ ∈ ꟽCl$\hbar_\alpha$ (Ą). But, ϰ ∉Ą. when ϰ ∈ ꟽD$\hbar_\alpha$ (Ą). So, ꟽD$\hbar_\alpha$ (Ą) ⊆ ꟽCl$\hbar_\alpha$ (Ą) – Ą ... (1)

If ϰ ∈ ꟽCl$\hbar_\alpha$ (Ą) – Ą , ϰ ∈ ꟽCl$\hbar_\alpha$ (Ą) and ϰ ∉ Ą ,∀ is ꟽ$\hbar_\alpha$ – *os* G containing ϰ, thus G ∩ ꟽCl$\hbar_\alpha$ (Ą) ≠ φ ∋ G ∩ (ꟽCl$\hbar_\alpha$ (Ą) –{ϰ})≠ φ ⇒ ϰ ∈ ꟽD$\hbar_\alpha$ (Ą) . Therefore ꟽCl$\hbar_\alpha$ (Ą) – Ą⊆ ꟽD$\hbar_\alpha$ (Ą) … (2)

We get from (1) and (2) ꟽD$\hbar_\alpha$ (Ą)= ꟽCl$\hbar_\alpha$ (Ą) – Ą.

**Definition 3.16** A subset Ą of (Ủ, $\text{Ƭ}_R$ (Ӿ)),the class $\text{Ƭ}_R^{\hbar_\alpha}$ (Ӿ) of all intersection topology on Ą is called a Nano-subspace of (Ủ, $\text{Ƭ}_R^{\hbar_\alpha}$ (Ӿ)) denoted by $\text{Ƭ}_R^{*\hbar_\alpha}$ (Ӿ).

**Example 3.17** Let Ủ ={2,3,4}, Ủ/ Ʀ={{2},{3},{4}} and Ӿ={2,3}, then $\text{Ƭ}_R$ (Ӿ) = { φ, Ủ,{2,3}}, we get $\text{Ƭ}_R^{\hbar_\alpha}$ (Ӿ) = { φ, Ủ ,{2} ,{3},{4},{2,3},{2,4},{3,4}}. Since Ą ={2,3}, thus $\text{Ƭ}_R^{*\hbar_\alpha}$ (Ӿ) ={ φ, Ą,{2},{3},{2,3}}.

**Remark 3.18**

Explains the relation between some types of Nano open sets in ꟽƬS.

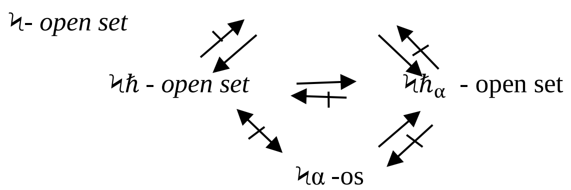


**Figure (1): The relation between some types of Nano open sets in ꟽƬS**

## IV. ꟽ$\hbar_A$ - CONTINUITY MAPPING

**Definition 4.1** Let ƒ: (Ủ, $\text{Ƭ}_R$ (Ӿ)) →( V, $\text{Ƭ}_R$\` (Ҷ)) is said to be Nano$\hbar_\alpha$ - continuous Mapping (ꟽ$\hbar_\alpha$ *-cont*) if $ƒ^{-1}$(Ą) is Nano ꟽ$\hbar_\alpha$ - *os* in Ủ, for every ꟽ - *os* in V.

**Example 4.2** Let ƒ: (Ủ, $\text{Ƭ}_R$ (Ӿ)) →(Ủ, $\text{Ƭ}_R$\` (Ҷ)) , let Ủ={a,b,c}, Ủ/Ʀ= {{a},{b},{c}}, when Ӿ={a,c}, then we get $\text{Ƭ}_R$ (Ӿ) = { φ, Ủ,{a,c}} and $\text{Ƭ}_R^{\hbar_\alpha}$(Ӿ) = Ƥ(Ủ). and V= {1,2,4}, V/Ʀ = {{1},{2},{4}} when Ҷ= {1,4}, then we get $\text{Ƭ}_R$\` (Ҷ) = {φ, V, {1,4}}. Define the mapping by ƒ (a) =1, ƒ (b) = 4, ƒ (c) = 2. Therefore, ƒ is ꟽ$\hbar_\alpha$ - *cont* mapping.

**Propositions 4.3**

**1**. Every ꟽ$\hbar$ - *cont* mapping is ꟽ$\hbar_\alpha$ - *cont* mapping.

**Proof.**

Let ƒ: (Ủ, $\text{Ƭ}_R$ (Ӿ)) →( V, $\text{Ƭ}_R$\` (Ҷ)) be mapping , since ƒ is ꟽ$\hbar$ - *cont*, then $ƒ^{-1}$ (Ą) is ꟽ$\hbar$ *-os* in Ủ for every Ą ꟽ*-os* in V, By the proposition (3.5 (1)) , we get every ꟽ$\hbar$ *-os* is ꟽ$\hbar_\alpha$ - *os* , then $ƒ^{-1}$(Ą) is ꟽ$\hbar_\alpha$ - *os* in Ủ. Hence ƒ is ꟽ$\hbar_\alpha$ - *cont.*

**2**. Every ꟽ - *cont* mapping is ꟽ$\hbar_\alpha$ - *cont* mapping.

**Proof.**

By using the definition of Nano continuous and by the proposition (3.5(2)), then every ꜧ - *cont* mapping is $ꜧ\hbar_\alpha$ *cont* mapping.

The converse of this proposition is not necessary to be true.

**Example 4.4** Let ƒ: (Ư, $Ƭ_R$ (Ӿ)) →( V, $Ƭ_{R}$` (Ҷ)) , Ư={a,b,c,d}, Ư/Ꞧ ={{a},{b},{c},{d}} and Ӿ={a}.So, $Ƭ_R$ (Ӿ) = {φ, Ư ,{a}} ,$Ƭ_R^{\hbar}$ (Ӿ) ={φ, Ư,{a},{b,c,d}} ,we get $Ƭ_R^{\alpha}$ (Ӿ) ={ φ, Ư, {a} ,{a,b},{a,c}, ,{a,d},{a,b,c},{a,b,d}, {a,c,d}},$Ƭ_R^{\hbar_\alpha}$ (Ӿ) = Þ(Ư) . Let V={1,2,3} , V/Ꞧ ={{1},{2} ,{3}} and Ҷ={1,3} ,$Ƭ_R$` (Ҷ) ={ φ, Ư,{1,3}} .Define the mapping by ƒ (a) =2, ƒ (b) = 3, ƒ (c) = 1 , thus ƒ is $ꜧ\hbar_\alpha$ - *cont* but it is not ($ꜧ\hbar$ - *cont*, ꜧ - *cont*) , (because $ƒ^{-1}$({1,3}) ={b,c}∉ $Ƭ_R^{\hbar}$ (Ӿ) and $Ƭ_R$ (Ӿ)).

**Definition 4.5** Let (Ư, $Ƭ_R$ (Ӿ)) and (V, $Ƭ_R$` (Ҷ)) are two ꜧƬ$\mathcal{S}$ , then ƒ: (Ư, $Ƭ_R$ (Ӿ)) →(V, $Ƭ_R$` (Ҷ)) is called Nano $\hbar_\alpha$ – totally continuous ( $ꜧΓ\hbar_\alpha$ – *cont*) mapping if the inverse image for the all $ꜧ\hbar_\alpha$ - os in V is ꜧ- clopen set in Ư.

**Example 4.6**

A mapping ƒ: (Ư, $Ƭ_R$ (Ӿ)) → (V, $Ƭ_R$` (Ҷ)). Let Ư =V ={a,b,c}, Ư/Ꞧ ={{a}, {b,c}} when Ӿ={a,b}, then $Ƭ_R$ (Ӿ) = { φ, Ư,{a},{b,c}}. Let V/Ꞧ ={{a}, {b},{c}} when Ҷ={b,c}, then $Ƭ_R$ ` (Ҷ) ={ φ, V,{b,c}}and $Ƭ_R^{\hbar_\alpha}$`(Ҷ)= Þ(V). Define the mapping by ƒ(a)= ƒ(b)= ƒ(c)={a}, thus ƒ is ($ꜧΓ\hbar_\alpha$ – *cont*) mapping.

**Proposition 4.7**

A mapping ƒ: (Ư, $Ƭ_R$ (Ӿ)) → (V, $Ƭ_R$` (Ҷ)), then:

**1**. Every $ꜧΓ\hbar_\alpha$ – *cont* is $ꜧΓ\hbar$ –*cont.*

**Proof:**

To prove ƒ is $ꜧΓ\hbar$ –*cont*. Let Ą is $ꜧ\hbar$ - *os*. By using preposition (3.5(1)) (∀ $ꜧ\hbar$ - *os* is $ꜧ\hbar_\alpha$ – *os*) and we get Ą is $ꜧ\hbar_\alpha$ – *os*. Since ƒ: (Ư, $Ƭ_R$ (Ӿ)) →(V, $Ƭ_R$` (Ҷ)) is $ꜧΓ\hbar_\alpha$ -*cont*. Then $ƒ^{-1}$ (Ą) is Nano clepen in Ư. Thus, ƒ is $ꜧΓ\hbar$-*cont.*

The convers is not need to be true. By the following example:

Let ƒ: (Ư, $Ƭ_R$ (Ӿ)) →( V, $Ƭ_R$` (Ҷ)) and let V=Ư={a,b,c,d}, Ư/Ꞧ ={{a} ,{b,c},{d}} ,when Ӿ={a,b}and we get $Ƭ_R$ (Ӿ) = {φ, Ư,{a}, {b,c,d}}, V/ Ꞧ ={{a},{b, c},{d}} and Ӿ={a}. So, $Ƭ_R$` (Ҷ)= {φ,Ư,{a}} and $Ƭ_R^{\hbar}$`( Ҷ) ={φ, Ư,{a} ,{b,c,d}},$Ƭ_R^{\hbar_\alpha}$`(Ҷ) = Þ(V). Define ƒ by identity mapping, then ƒ it is $ꜧΓ\hbar$ –*cont* mapping but it is not $ꜧΓ\hbar_\alpha$ – *cont* mapping.

**2**. Every $ꜧΓ\hbar_\alpha$ – *cont* is ꜧΓ- *cont.*

**Proof.**

To prove ƒ is ꜧΓ-*cont*. Let Ą is ꜧ - *os*. By using preposition (3.5 (2)) (∀ ꜧ - *os* is $ꜧ\hbar_\alpha$ – *os*) and we get Ą is $ꜧ\hbar_\alpha$ – *os*. Since ƒ: (Ư, $Ƭ_R$ (Ӿ)) →( V, $Ƭ_R$` (Ҷ)) is $ꜧΓ\hbar_\alpha$ – *cont* . Then $ƒ^{-1}$ (Ą) is Nano clopen in Ư. Thus, ƒ is ꜧΓ-*cont.*

The convers is not need to be true. By the following example:

Let ƒ: (Ư, $Ƭ_R$ (Ӿ)) →( V, $Ƭ_R$` (Ҷ)) and let V=Ư={a,b,c}, Ư/Ꞧ={{a},{b,c}} ,when Ӿ={a,b}and we get $Ƭ_R$ (Ӿ) = {φ, Ư,{a},{b,c}},V/Ꞧ={{a},{b},{c}} ,when Ҷ={b,c}, we get $Ƭ_R$ ` (Ҷ) ={ φ, V,{b,c}} and, $Ƭ_R^{\hbar_\alpha}$`(Ҷ) = Þ(V) . Define ƒ by ƒ(a) = a, ƒ(b) = c , ƒ(c) = b .Then ƒ it is ꜧΓ- *cont* mapping but it is not $ꜧΓ\hbar_\alpha$-*cont* mapping.(because $ƒ^{-1}$({c}) ={b} is not Nano clopen set).

**Remark 4.8** The two mappings $ꜧΓ\hbar_\alpha$ – *cont* and $ꜧ\hbar_\alpha$ - *cont* are independent mappings.

**Definition 4.9** A map ƒ: (Ư, $Ƭ_R$ (Ӿ)) →( V, $Ƭ_R$` (Ҷ)), Ą⊆ Ư, then:

**1.** Nano $\hbar_\alpha$ - open mapping ($ꜧ\hbar_\alpha$ -OM) ,if ƒ(Ą) is $ꜧ\hbar_\alpha$ - *os* in V, for every Ą is ꜧ- *os* in Ư .

**2.** Nano $\hbar_\alpha$ -closed map ($ꜧ\hbar_\alpha$ -CM), if ƒ(Ą) is $ꜧ\hbar_\alpha$ - *cs* in V, for every Ą is ꜧ- *cs* in Ư.

**Examples 4.10**

**1.** From the example (4.2), then ,$Ƭ_R$` (Ҷ) ={ φ, V,{1,4}} and $Ƭ_R^{\hbar_\alpha}$`(Ҷ) =Þ(V) , we get ƒ(φ) = φ ∈ $Ƭ_R^{\hbar_\alpha}$`(Ҷ) ƒ(Ư) = V∈ $Ƭ_R^{\hbar_\alpha}$`(Ҷ) , ƒ({a,c}) ={1,2} ∈ $Ƭ_R^{\hbar_\alpha}$`(Ҷ) Therefore, ƒ is $ꜧ\hbar_\alpha$ - OM.

**2.** From the example (4.2). Since $Ƭ_R$ (Ӿ) = {φ, Ư, {a,c}} ,($Ƭ_R$ (Ӿ)) $^c$ ={ φ, Ư,{b}}.Therefore, ƒ is $ꜧ\hbar_\alpha$ - CM .

**Propositions 4.11**

A mapping ƒ: (Ư, $Ƭ_R$ (Ӿ)) → (V, $Ƭ_R$` (Ҷ)), then:

**1.** ƒ is ꜧ-*OM*, then ƒ is $ꜧ\hbar_\alpha$ - OM.

**Proof.**

It's an obvious proof.

Converse of this proposition is not necessary to be true. From the example (4.2) (we get ƒ is $ꜧ\hbar_\alpha$ - OM but it is not ꜧ-*OM*).

2. Every $ꜧ\hbar - OM$ is $ꜧ\hbar_\alpha$ - OM.

**Proof.**

By using proposition (3.5 (1)).

**Definition 4.12** A bijective map ƒ: (Ư, $Ƭ_R$ (Ӿ)) →( V, $Ƭ_R$` (Ҷ)) is said to be Nano$\hbar_\alpha$ - homeomorphism if ƒ is $ꜧ\hbar_\alpha$ - *cont*. and ƒ is $ꜧ\hbar_\alpha$ - OM .

From the example (4.2) is a bijective map and since the map is $ꜧ\hbar_\alpha$ - *cont* and $ꜧ\hbar_\alpha$ - OM. Thus, ƒ is $ꜧ\hbar_\alpha$ - homeomorphism.

**Proposition 4.13**

A map ƒ: (Ư, $Ƭ_R$ (Ӿ)) →( V, $Ƭ_R$` (Ҷ)), then:

**1.** If ƒ is ꜧ- homeomorphism, then it is $ꜧ\hbar_\alpha$ - homeomorphism. But the convers is not necessary to be true.

**Proof.**

Since (every ꜧ- *cont* is $ꜧ\hbar_\alpha$ - *cont*) and (every ꜧ-*OM* is $ꜧ\hbar_\alpha$ -OM). Thus, ƒ is $ꜧ\hbar_\alpha$ - homeomorphism.

2. Every $ꜧ\hbar$ - homeomorphism is $ꜧ\hbar_\alpha$ - homeomorphism. But the convers is not necessary to be true.

**Proof.**

Since (every $ꜧ\hbar$ - *cont* is $ꜧ\hbar_\alpha$ - *cont*) and (every $ꜧ\hbar$ -OM is $ꜧ\hbar_\alpha$ - OM) Thus, ƒ is $ꜧ\hbar_\alpha$ - homeomorphism.

**Remark 4.14**

Let ƒ: (Ư, $Ƭ_R$ (Ӿ)) and (V, $Ƭ_R$` (Ҷ)) is said to be $ꜧ\hbar_\alpha$ - homeomorphic if there exist $ꜧ\hbar_\alpha$ - homeomorphism

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from (U̇, $Ƭ_R$ (Ӿ)) to (V̇, $Ƭ_R$` (Ч)) is denoted by (U̇, $Ƭ_R$ (Ӿ)) ≅ (V̇, $Ƭ_R$` (Ч)).

## V. CONCLUSION

**The main results of this paper are stated as below:**

1. Every ꝁ-open set in any ꝁƬ$\mathcal{S}$ (U̇, $Ƭ_R$ (Ӿ)) is ꝁ$\hbar_\alpha$ - *open set.*
2. Every ꝁ$\hbar$ - open set in any ꝁƬ$\mathcal{S}$ (U̇, $Ƭ_R$ (Ӿ)) is ꝁ$\hbar_\alpha$ - open set.
3. Every ꝁα - open set in any ꝁƬ$\mathcal{S}$ (U̇, $Ƭ_R$ (Ӿ)) is ꝁ$\hbar_\alpha$ - *open set*.
4. Every ꝁ$\hbar$ - *cont* mapping is ꝁ$\hbar_\alpha$ - *cont* mapping.
5. Every ꝁ- *cont* mapping is ꝁ$\hbar_\alpha$ - *cont* mapping.
6. Every ꝁ-*OM* (ꝁ$\hbar$ -OM) is ꝁ$\hbar_\alpha$ - OM.

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